Homework Feedback 4

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**P. 357 #8** Gauss-Jordan method, which eliminates the backward substitution in Gauss-elimination method.

**Answer:** 1. Change the loop index in step 4 -> Step 4: for j = 1, 2, …, n if (j!=i) then do the elimination in step 5 and 6.

2. Chang step 8,9 to remove the backward substitution.

**P. 358 #11** a. show that the Gauss-Jordan method requires multiplications/divisions and additions/subtractions.

b. Make a table comparing the required operations for the Gauss-Jordan and Gaussian elimination methods for n = 3, 10,50, 100. Which method requires less computation?

**Answer:**

a. For multiplication/division, it is as follows:

Divisions when computing x

Divisions when computing m

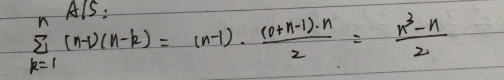
Multiplications when performing elimination

For addition/subtractions, it is as follows:

b. Just use the formulas in the answers to question **a** to compute the number of operations in Gauss-Jordan and Gauss elimination methods.

**Typical errors:**

1. There exist errors in the derivation of the number of arithmetic operations in Gauss-Jordan method. For example, for additions/subtractions, the following derivation is incorrect:



Gauss-Jordan是否比Gauss-elimination稳定？

在求解稳定性的意思是指当矩阵A或者b带有误差，求出来的解x的误差应该是可控的，即不会因为A或者b中的微小误差或者求解时的舍入误差导致x的误差非常大，所求的解无法使用。理论上只能说明Gauss-Jordan和Gauss-elimination的误差是在可比范围内。另外，Gauss-Jordan可用于求解矩阵逆矩阵。如果对Gauss-Jordan的稳定性分析感兴趣，可以阅读参考文献： G. Peters, J.H. Wilkinson, On the Stability of Gauss-Jordan Elimination with Pivoting, Communication of ACM, Volume 18, Issue 1, 1975。

**P. 397 #7** a. Show that the LU Factorization Algorithm requires multiplications /divisions and additions/subtractions.

b. Show that solving Ly = b, where L is a lower-triangular matrix with for all i,

requires multiplications/divisions and additions/subtractions.

c. Show that solving Ax = b by first factoring A into A = LU and then solving Ly = b and

U x = y requires the same number of operations as the Gaussian Elimination Algorithm 6.1.

d. Count the number of operations required to solve m linear systems for k = 1, ... ,m by first factoring A and then using the method of part (c) m times.

**Answer:**

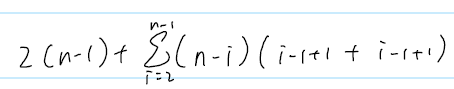
1. Count the number of arithmetic operations in Algorithm 6.4 in the textbook.

For muliplications/divisions, we have: step 2: n-1 step 3,4,5: ,step 6: n-1. Sum all these number together to obtain the answer . Similarly, we can get the number of additions/subtractions for LU factorization.

1. With , all the divisions by do not need to be counted. Therefore, the number of multiplications/divisions is . Similar results can be computed for additions/subtractions.
2. Sum all the numbers above we prove the result.
3. Sum all the computations required we get the result.

**Typical errors:**

1. Derivation errors. For example, for multiplications/divisions, the following formula is incorrect:



Since , we do need to count the division at .

**P. 397 #7** let

Find all values ofand for which:

1. A is singular.
2. A is strictly diagonally dominant.
3. A is symmetric.
4. A is positive definite.

**Answer:**

a. If A is singular, then det(A) = 0, thus, we get: . For alland satisfy such equation, A is singular.

b. If A is strictly diagonally dominant, we have .

c. If A is symmetric, .

d. If A is positive definite, then obviously. According to the properties of the positive definite matrix, the determinant of every leading principal sub-matrix should be greater than 0 (NA04\_ch6\_B.ppt, slide 10), (note the answer to question a, it means 0 so that the determinant of the matrix is greater than 0). We can verify that A with such value also satisfies other properties of the positive definite matrix.

**Typical errors:**

1. Incorrect results for d, for examples:

